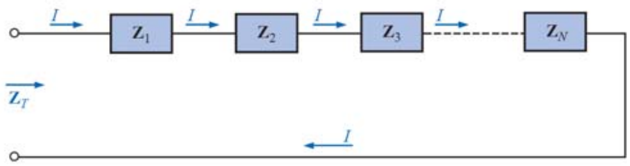
Circuitos Serie y Paralelo en AC.

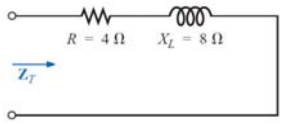
Dpto. de Física. ~~Facultad de Ciencias Físico-Mat. y Nat.~~ (UNSL)

Impedancias en Serie:



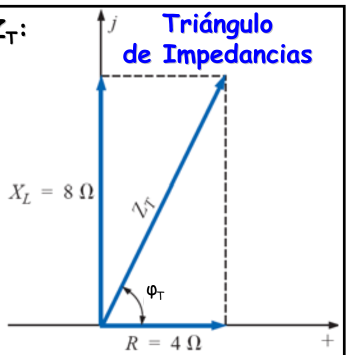
$$\boxed{\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N}$$

Ejemplo: Determine la Z_T:

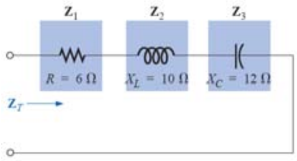


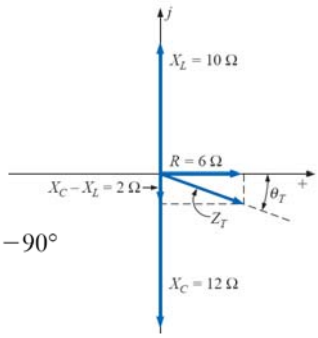
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$
$$= R\ \angle 0° + X_L\ \angle 90°$$
$$= R + jX_L = 4\ \Omega + j8\ \Omega$$
$$\mathbf{Z}_T = \mathbf{8.944\ \Omega\ \angle 63.43°}$$

Triángulo de Impedancias



$$|Z_T| = \sqrt{R^2 + X_L^2}$$
$$\varphi_T = Arctg\,\frac{X_L}{R}$$



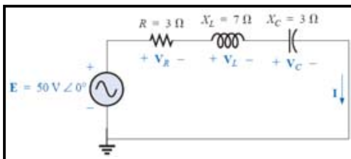


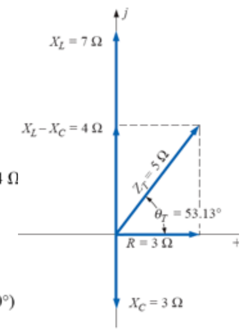
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$
$$= R\ \angle 0° + X_L\ \angle 90° + X_C\ \angle -90°$$
$$= R + jX_L - jX_C$$
$$= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega$$
$$\mathbf{Z}_T = \mathbf{6.325\ \Omega\ \angle -18.43°}$$

Ley de Ohm en AC:

$$I = \frac{V}{Z}$$

OJO!!! I y V son FASORES!!!





$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R\ \angle 0° + X_L\ \angle 90° + X_C\ \angle -90°$$
$$= 3\ \Omega + j\,7\ \Omega - j\,3\ \Omega = 3\ \Omega + j\,4\ \Omega$$

$$\boxed{\mathbf{Z}_T = \mathbf{5\ \Omega\ \angle 53.13°}}$$

$$\boxed{\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\ \text{V}\ \angle 0°}{5\ \Omega\ \angle 53.13°} = \mathbf{10\ A\ \angle -53.13°}}$$

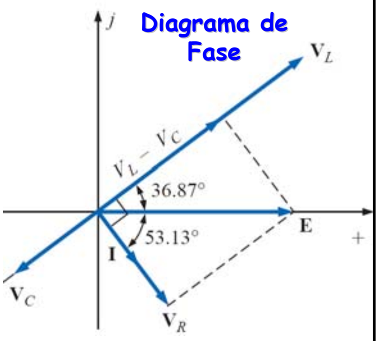
$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\ \angle\theta)(R\ \angle 0°) = (10\ \text{A}\ \angle -53.13°)(3\ \Omega\ \angle 0°)$$
$$= \mathbf{30\ V\ \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\ \angle\theta)(X_L\ \angle 90°) = (10\ \text{A}\ \angle -53.13°)(7\ \Omega\ \angle 90°)$$
$$= \mathbf{70\ V\ \angle 36.87°}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\ \angle\theta)(X_C\ \angle -90°) = (10\ \text{A}\ \angle -53.13°)(3\ \Omega\ \angle -90°)$$
$$= \mathbf{30\ V\ \angle -143.13°}$$

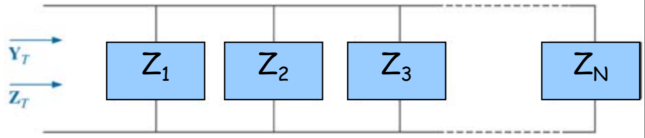
$$V = 50\angle 0^\circ$$
$$I = 10\angle -53.13^\circ$$
$$V_R = 30\angle -53.13^\circ$$
$$V_L = 70\angle 36.87^\circ$$
$$V_C = 30\angle -143.13^\circ$$

Diagrama de Fase



$$\Sigma_C \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L - \mathbf{V}_C = 0$$

Circuitos Paralelos en AC - Admitancia (Y)



$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \cdots \frac{1}{Z_N}$$

$$\boxed{Z = 1/Y}$$

$$Y_T = Y_1 + Y_2 + Y_3 + \ldots + Y_N$$

Conductancia y Susceptancia:

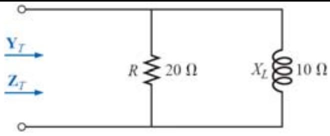
$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L\angle 90^\circ} = \frac{1}{X_L}\angle -90^\circ \qquad B_L = \frac{1}{X_L} \quad \text{(siemens, S)}$$

$$\mathbf{Y}_L = B_L\angle -90^\circ$$

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C\angle -90^\circ} = \frac{1}{X_C}\angle 90^\circ \qquad B_C = \frac{1}{X_C} \quad \text{(siemens, S)}$$

$$\mathbf{Y}_C = B_C\angle 90^\circ$$



$$\mathbf{Y}_R = G\angle 0^\circ = \frac{1}{R}\angle 0^\circ = \frac{1}{20\ \Omega}\angle 0^\circ$$
$$= \mathbf{0.05\ S\angle 0^\circ = 0.05\ S} + j\,\mathbf{0}$$

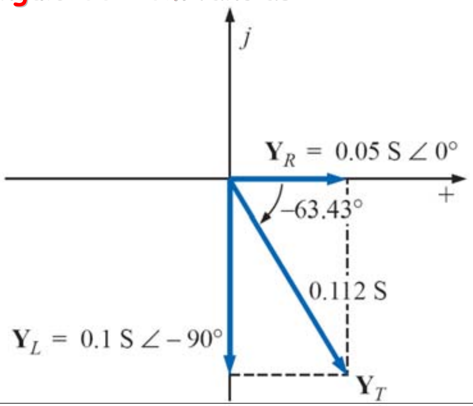
$$\mathbf{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10\ \Omega}\angle -90^\circ = \mathbf{0.1\ S\angle -90^\circ = 0} - j\,\mathbf{0.1\ S}$$

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ \text{S} + j\,0) + (0 - j\,0.1\ \text{S}) = \mathbf{0.05\ S} - j\,\mathbf{0.1\ S} = G - jB_L$$

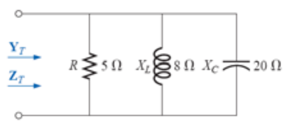
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.05\ \text{S} - j\,0.1\ \text{S}} = \frac{1}{0.112\ \text{S}\angle -63.43^\circ} = \mathbf{8.93\ \Omega\angle 63.43^\circ}$$

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega\angle 0^\circ)(10\ \Omega\angle 90^\circ)}{20\ \Omega + j\,10\ \Omega} = \frac{200\ \Omega\angle 90^\circ}{22.36\angle 26.57^\circ} = \mathbf{8.93\ \Omega\angle 63.43^\circ}$$

Triángulo de Admitancias:



$$\mathbf{Y}_R = G\angle 0^\circ = \frac{1}{R}\angle 0^\circ = \frac{1}{5\ \Omega}\angle 0^\circ$$
$$= \mathbf{0.2\ S\angle 0^\circ = 0.2\ S} + j\,\mathbf{0}$$



$$\mathbf{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{8\ \Omega}\angle -90^\circ$$
$$= \mathbf{0.125\ S\angle -90^\circ = 0} - j\,\mathbf{0.125\ S}$$

$$\mathbf{Y}_C = B_C\angle 90^\circ = \frac{1}{X_C}\angle 90^\circ = \frac{1}{20\ \Omega}\angle 90^\circ = \mathbf{0.050\ S\angle +90^\circ = 0} + j\,\mathbf{0.050\ S}$$

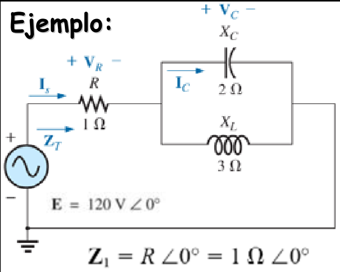
$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$$
$$= (0.2\ \text{S} + j\,0) + (0 - j\,0.125\ \text{S}) + (0 + j\,0.050\ \text{S})$$
$$= 0.2\ \text{S} - j\,0.075\ \text{S} = \mathbf{0.2136\ S\angle -20.56^\circ}$$

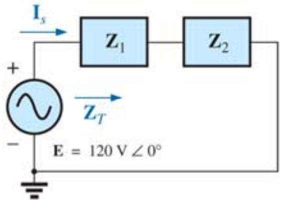
$$\boxed{\mathbf{Z}_T = \frac{1}{0.2136\ \text{S}\angle -20.56^\circ} \quad \boxed{= \mathbf{4.68\ \Omega\angle 20.56^\circ}}}$$

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C} = \frac{(5\ \Omega\angle 0^\circ)(8\ \Omega\angle 90^\circ)(20\ \Omega\angle -90^\circ)}{(5\ \Omega\angle 0^\circ)(8\ \Omega\angle 90^\circ) + (8\ \Omega\angle 90^\circ)(20\ \Omega\angle -90^\circ) + (5\ \Omega\angle 0^\circ)(20\ \Omega\angle -90^\circ)}$$

$$= \frac{800\ \Omega\angle 0^\circ}{40\angle 90^\circ + 160\angle 0^\circ + 100\angle -90^\circ} \boxed{= \mathbf{4.68\ \Omega\angle 20.56^\circ}}$$

Ejemplo:





$$\mathbf{Z}_1 = R\angle 0^\circ = 1\ \Omega\angle 0^\circ$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C\angle -90^\circ)(X_L\angle 90^\circ)}{-j X_C + j X_L} = \frac{(2\ \Omega\angle -90^\circ)(3\ \Omega\angle 90^\circ)}{-j\,2\ \Omega + j\,3\ \Omega}$$

$$= \frac{6\ \Omega\angle 0^\circ}{j\,1} = \frac{6\ \Omega\angle 0^\circ}{1\angle 90^\circ} = 6\ \Omega\angle -90^\circ$$

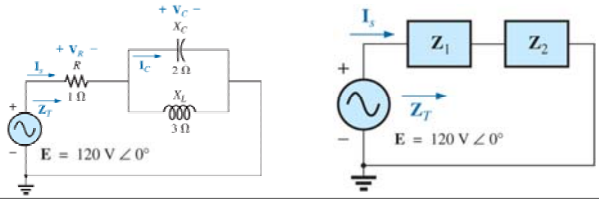
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\,6\ \Omega = \mathbf{6.08\ \Omega\angle -80.54^\circ}$$

$$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\ \text{V}\angle 0^\circ}{6.08\ \Omega\angle -80.54^\circ} = \mathbf{19.74\ A\angle 80.54^\circ}$$

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\ \text{A}\angle 80.54^\circ)(1\ \Omega\angle 0^\circ) = \mathbf{19.74\ V\angle 80.54^\circ}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\ \text{A}\angle 80.54^\circ)(6\ \Omega\angle -90^\circ) = \mathbf{118.44\ V\angle -9.46^\circ}$$

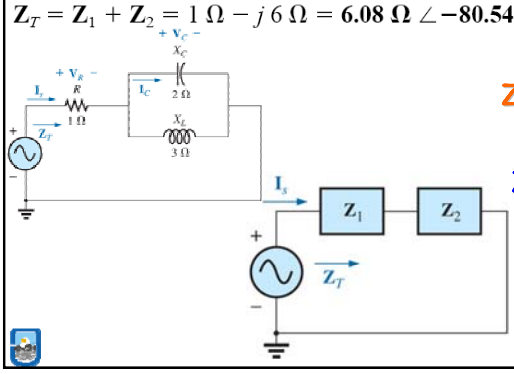
$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\ \text{V}\angle -9.46^\circ}{2\ \Omega\angle -90^\circ} = \mathbf{59.22\ A\angle 80.54^\circ}$$

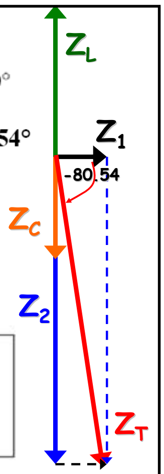


$$\mathbf{Z}_1 = R\angle 0^\circ = 1\ \Omega\angle 0^\circ$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(2\ \Omega\angle -90^\circ)(3\ \Omega\angle 90^\circ)}{-j\,2\ \Omega + j\,3\ \Omega} = 6\ \Omega\angle -90^\circ$$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\,6\ \Omega = \mathbf{6.08\ \Omega\angle -80.54^\circ}$$





$\mathbf{I}_s = \mathbf{19.74\ A\angle 80.54^\circ}$

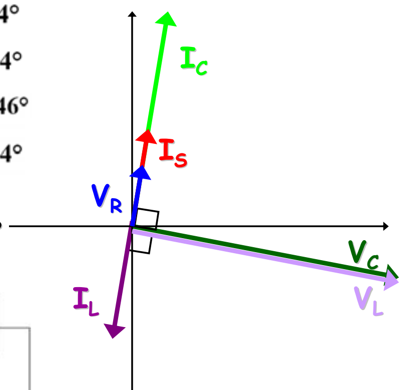
$\mathbf{V}_R = \mathbf{19.74\ V\angle 80.54^\circ}$

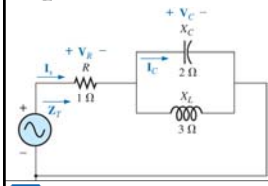
$\mathbf{V}_C = \mathbf{118.44\ V\angle -9.46^\circ}$

$\mathbf{I}_C = \mathbf{59.22\ A\angle 80.54^\circ}$

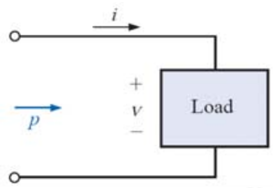
$\mathbf{I_L = 39.48\angle -99.46^\circ}$

$\mathbf{V_L = 118.44\angle -9.46^\circ}$





Potencia en AC.



$$p(t)=v(t)i(t)$$

$$v = V_m \sin(\omega t + \theta)$$

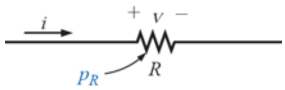
$$i = I_m \sin \omega t$$

$$p = V_m I_m \sin \omega t \sin(\omega t + \theta)$$

$$\boxed{p = VI\cos\theta(1 - \cos 2\omega t) + VI\sin\theta(\sin 2\omega t)}$$

Factor de Potencia: $F_p=\cos(\theta)$

Potencia en un circuito resistivo

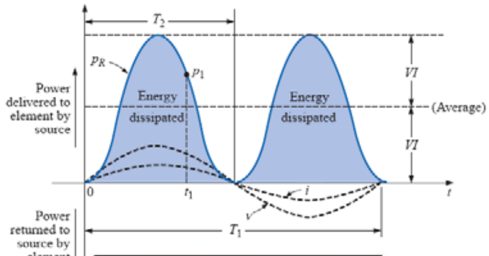


$$p = VI\cos\theta(1 - \cos 2\omega t) + VI\sin\theta(\sin 2\omega t)$$

$$p_R = VI\cos(0°)(1 - \cos 2\omega t) + VI\sin(0°)\sin 2\omega t$$
$$= VI(1 - \cos 2\omega t) + 0$$

$$p_R = VI - VI\cos 2\omega t$$

Potencia instantánea

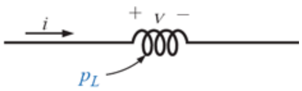


$$p_R = VI - VI\cos 2\omega t$$

$$P = VI = \frac{V_m I_m}{2} = I^2R = \frac{V^2}{R} \quad \text{(watts, W)}$$

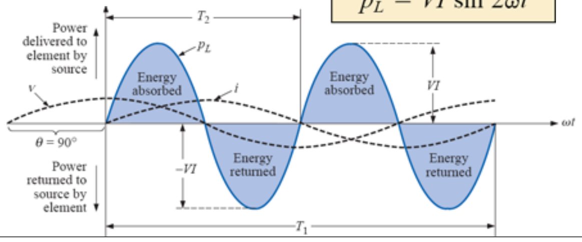
Potencia Promedio (Real)

Potencia en un circuito Inductivo



$$p_L = VI\cos(90°)(1 - \cos 2\omega t) + VI\sin(90°)(\sin 2\omega t)$$
$$= 0 + VI\sin 2\omega t$$

$$p_L = VI\sin 2\omega t$$



$$Q = VI\sin\theta \quad \text{(volt-ampere reactive, VAR)}$$

Potencia Reactiva

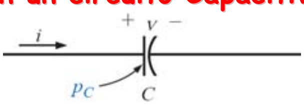
Para el Inductor:

$$Q_L = VI \quad \text{(VAR)}$$

$$Q_L = I^2X_L \quad \text{(VAR)}$$

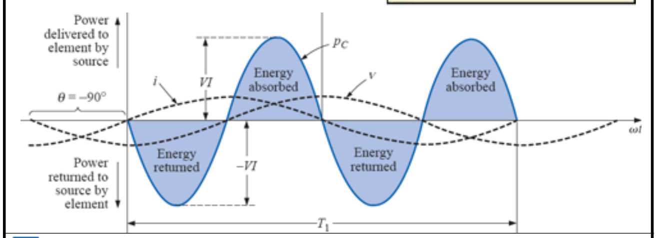
$$Q_L = \frac{V^2}{X_L} \quad \text{(VAR)}$$

Potencia en un circuito Capacitivo



$$p_C = VI\cos(-90°)(1 - \cos 2\omega t) + VI\sin(-90°)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

$$p_C = -VI\sin 2\omega t$$



$$Q = VI\sin\theta \quad \text{(volt-ampere reactive, VAR)}$$

Potencia Reactiva

Para el Capacitor:

$$Q_C = VI \quad \text{(VAR)}$$

$$Q_C = I^2X_C \quad \text{(VAR)}$$

$$Q_C = \frac{V^2}{X_C} \quad \text{(VAR)}$$

Triángulo de Potencias:

Las Potencias Reactivas y Real se relacionan:

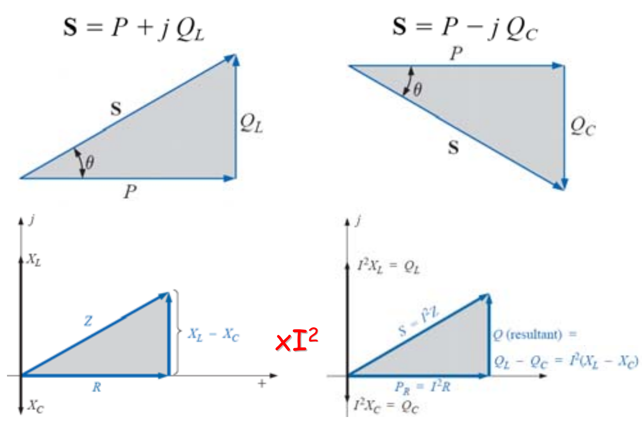
$$\boxed{\mathbf{S} = \mathbf{P} + \mathbf{Q}} \qquad \mathbf{S} = \mathbf{VI}$$

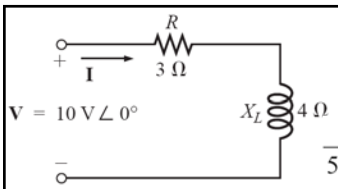
S es la Potencia Aparente. [VA]

$$\boxed{\mathbf{S} = \mathbf{P} + j(\mathbf{Q}_L - \mathbf{Q}_C)}$$

$$\mathbf{P} = P\,\angle 0^\circ \qquad \mathbf{Q}_L = Q_L\,\angle 90^\circ \qquad \mathbf{Q}_C = Q_C\,\angle -90^\circ$$

$$\mathbf{S} = \mathbf{VI}^*$$





$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\text{ V}\angle 0^\circ}{3\ \Omega + j\,4\ \Omega} = \frac{10\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = 2\text{ A}\angle -53.13^\circ$$

$$P = I^2R = (2\text{ A})^2(3\ \Omega) = 12\text{ W}$$

$$Q_L = I^2X_L = (2\text{ A})^2(4\ \Omega) = 16\text{ VAR }(L)$$

$$\mathbf{S} = P + j\,Q_L = 12\text{ W} + j\,16\text{ VAR }(L) = 20\text{ VA}\angle 53.13^\circ$$

$$\mathbf{S} = \mathbf{VI}^* = (10\text{ V}\angle 0^\circ)(2\text{ A}\angle +53.13^\circ) = 20\text{ VA}\angle 53.13^\circ$$