Inductores y Capacitores

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R, L y C sólo dependen de factores geométricos y del material. No Dependen ni de V ni de I ni de q

$i(t)=(V_0/R)(1-e^{-t/\tau})$ $\quad i(t)=(V_0/R)e^{-t/\tau}$

$V_L(t)=V_0e^{-t/\tau}$ $\quad V_L(t)=-V_0e^{-t/\tau}$

$\tau = L/R$

$i(t)=(V_0/R)e^{-t/\tau}$ $\quad i(t)=-(V_0/R)e^{-t/\tau}$

$V_C(t)=V_0(1-e^{-t/\tau})$ $\quad V_C(t)=V_0e^{-t/\tau}$

$\tau=RC$

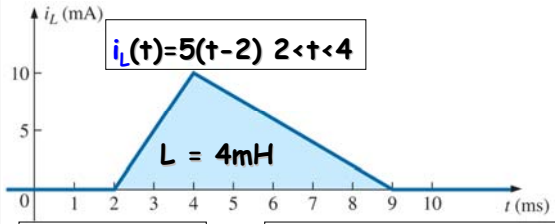
Estado Transitorio.

$$I = \frac{V_R}{R} \qquad V_R = IR$$

$$i_L(t) = \frac{1}{L}\int_{t=0}^{t=t} v_L(t)dt \qquad v_L(t) = L\frac{di(t)}{dt}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} \qquad v_C(t) = \frac{1}{C}\int_{t=0}^{t=t} i_C(t)dt$$

Válidas para todo tiempo



$i_L(t)=5(t-2)$ $2<t<4$

$i_L(t)=0$ $0<t<2$

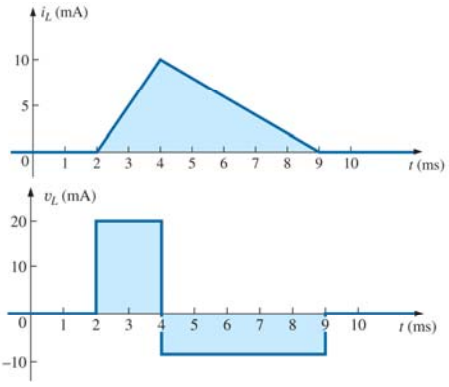
$i_L(t)=-2(t-4)+10$ $4<t<10$

$$v_L(t) = L\frac{di(t)}{dt}$$

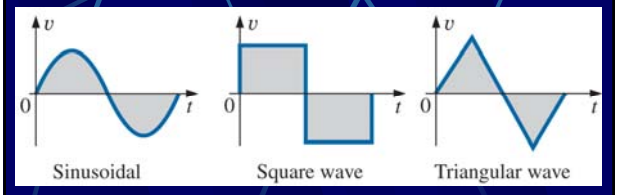
$v_L(t)=L5$ $2<t<4$

$v_L(t)=-L2$ $4<t<10$

Ejemplo

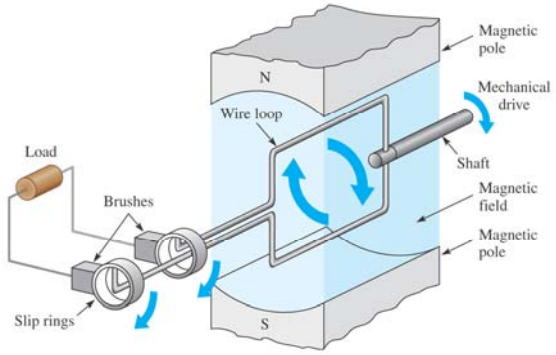


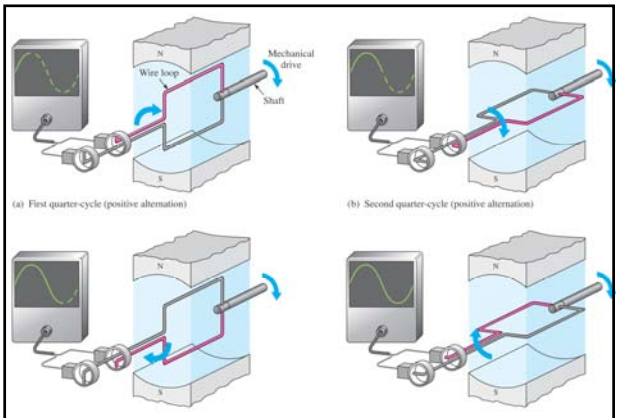
Corriente Alterna (AC)



Importancia:

Generador de Corriente Alterna



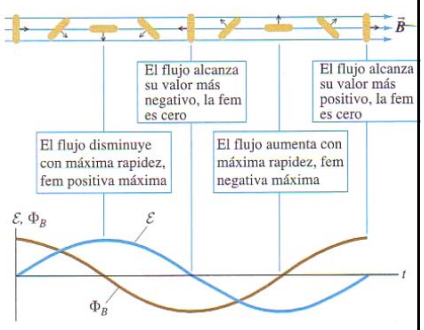


Flujo magnético:

$\Phi_B = BA\cos\theta$

Ley de Faraday:

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$



Flujo magnético:

$\Phi_B = BA\cos\theta$ $\quad \theta = \omega t$

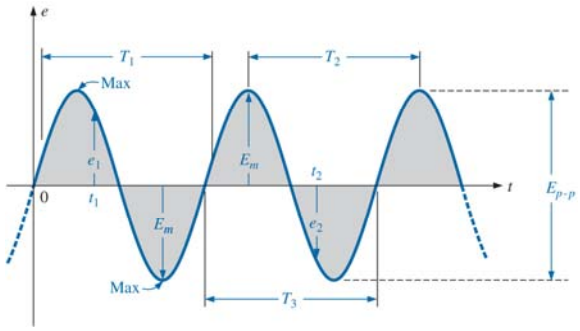
Ley de Faraday:

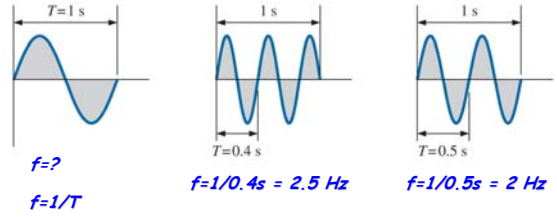
$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

$\varepsilon(t) = (NAB\omega)\operatorname{sen}(\omega t)$

$\varepsilon_{Máx} = NAB\omega$

$$\varepsilon(t) = \varepsilon_{Máx}\operatorname{sen}(\omega t)$$





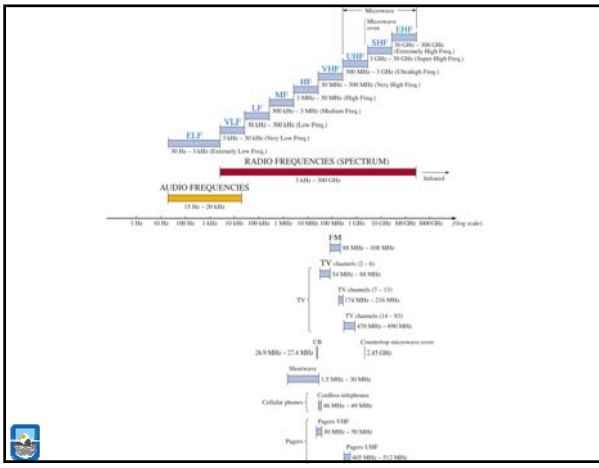
f=?

f=1/T

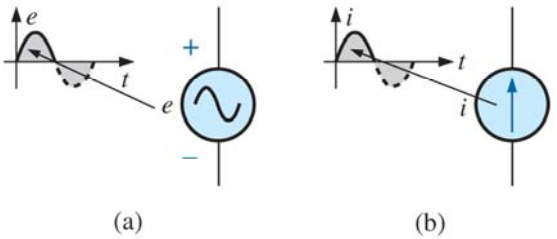
f=1/0.4s = 2.5 Hz

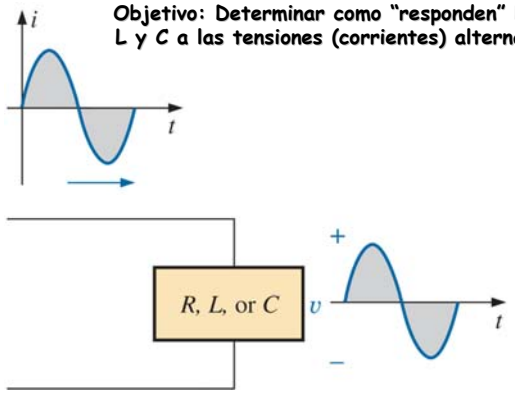
f=1/0.5s = 2 Hz

Frecuencia: Número de ciclos por segundo. Se Mide en [Hz]= 1/s



Polaridades:

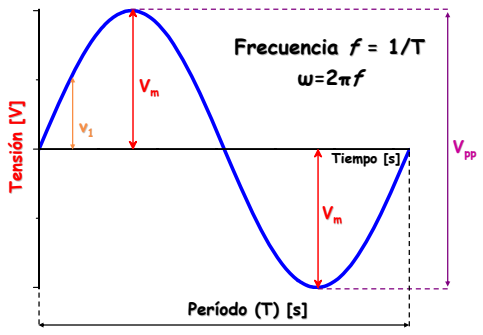




Objetivo: Determinar como "responden" R, L y C a las tensiones (corrientes) alternas

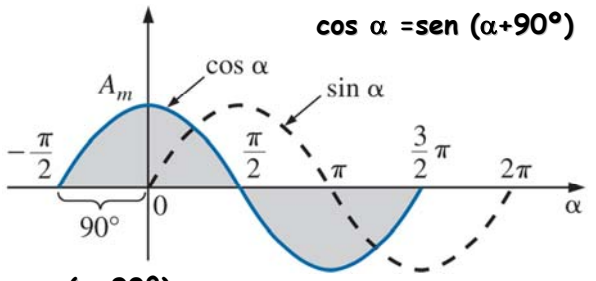
Que pasa con la forma de la respuesta?

Formato general: $v(t)=V_m sen(\omega t)$



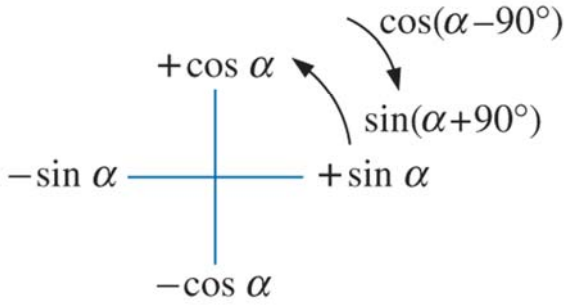
Fase:

$\cos\alpha = \text{sen}(\alpha+90°)$



$\cos(\alpha-90°) = \text{sen}\,\alpha$

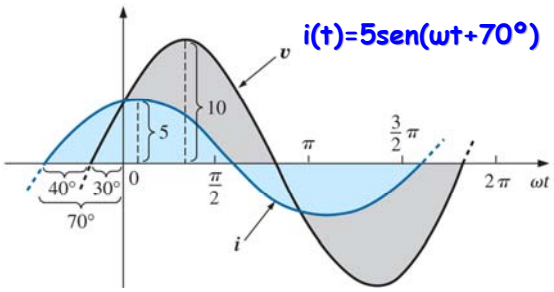
$v(t)=V_m\text{sen}(\omega t\pm\varphi)$



Ejemplo:

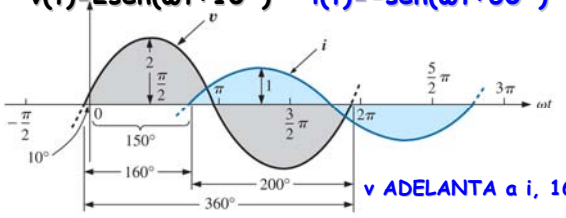
$v(t)=10\text{sen}(\omega t+30°)$

$i(t)=5\text{sen}(\omega t+70°)$



Diferencia de fase= 40°, i ADELANTA a v.

$v(t)=2\text{sen}(\omega t+10°)$ $i(t)=-\text{sen}(\omega t+30°)$



v ADELANTA a i, 160°

$i(t)=-\text{sen}(\omega t+30°)=\text{sen}(\omega t+30°-180°)=\text{sen}(\omega t-150°)$

v Adelanta a i, 160°

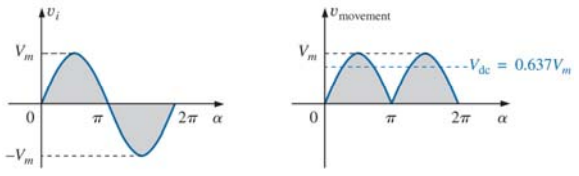
$i(t)=-\text{sen}(\omega t+30°)=\text{sen}(\omega t+30°+180)=\text{sen}(\omega t+210°)$

i Adelanta a v, 200°

Valor Medio:

$$y_{Medio}=\frac{1}{T}\int_0^T y(t)dt$$

$v(t)=V_m\text{sen}(\omega t\pm\varphi)$

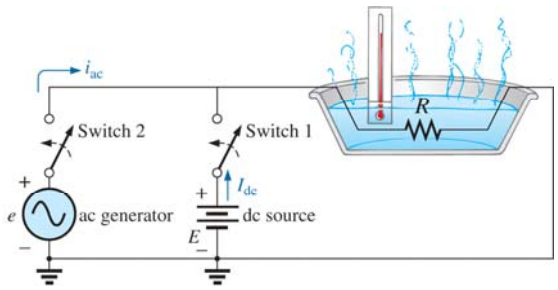


Valor Eficaz:

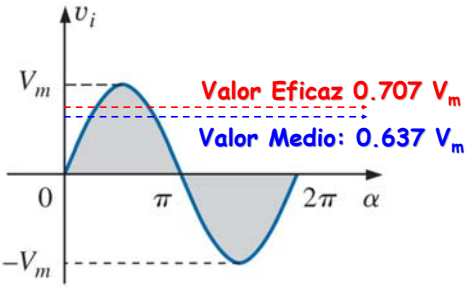
$$y_{Eficaz}=\sqrt{\frac{1}{T}\int_0^T y(t)^2\,dt}$$

$v(t)=V_m\text{sen}(\omega t\pm\varphi)$

$V_{eficaz}=V_m/(\sqrt{2}) = 0.707V_m$



Una corriente alterna tiene un valor EFICAZ de I Amperes si, al circular a través de la resistencia dada, origina en dicha resistencia una cantidad de calor media, por unidad de tiempo, igual al que produciría una corriente CONTINUA de valor I.



Una corriente alterna tiene un valor EFICAZ de I Amperes si, al circular a través de la resistencia dada, origina en dicha resistencia una cantidad de calor media, por unidad de tiempo, igual al que produciría una corriente CONTINUA de valor I.

Respuesta de R, L y C a la Corriente Alterna. Fasores.

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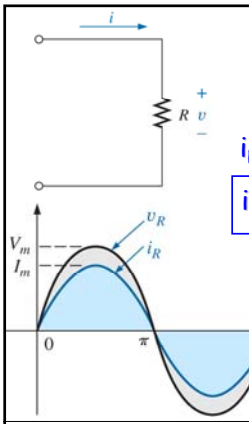


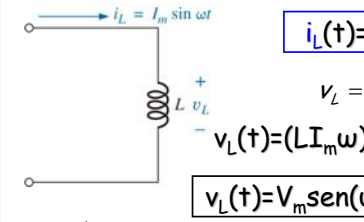
$v_R(t)=V_m\text{sen}(\omega t)$

Aplicando la Ley de Ohm: I=V/R

$i_R(t)=v_R(t)/R=(V_m/R)\text{sen}(\omega t)$

$i_R(t)=I_m\text{sen}(\omega t)$; $I_m=(V_m/R)$

Corriente y Tensión están en FASE en una Resistencia

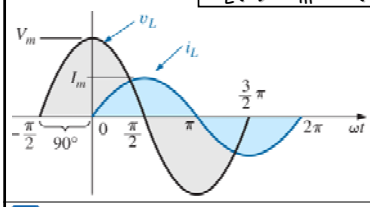


$i_L(t)=I_m\text{sen}(\omega t)$

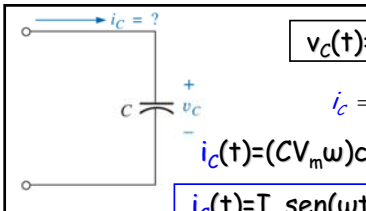
$v_L = L\frac{di_L(t)}{dt}$

$v_L(t)=(LI_m\omega)\cos(\omega t)$; $V_m=I_m(\omega L)$

$v_L(t)=V_m\text{sen}(\omega t+90°)$ $X_L=\omega L$



La Tensión adelanta 90° a la Corriente en un inductor

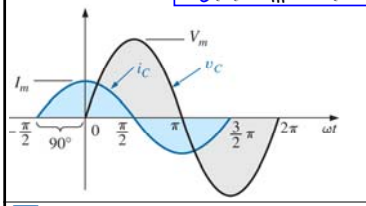


$v_C(t)=V_m\text{sen}(\omega t)$

$i_C = C\frac{dv_c(t)}{dt}$

$i_C(t)=(CV_m\omega)\cos(\omega t)$; $I_m=V_m(\omega C)$

$i_C(t)=I_m\text{sen}(\omega t+90°)$ $X_C=1/\omega C$



La Corriente adelanta 90° a la Tensión en un capacitor.

Ejemplo: Dada la corriente a través de un inductor de 0.1 H. Determine $v_L(t)$.

$i_L(t)=I_m\text{sen}(\omega t)$ $\quad v_L(t)=V_m\text{sen}(\omega t+90°)$

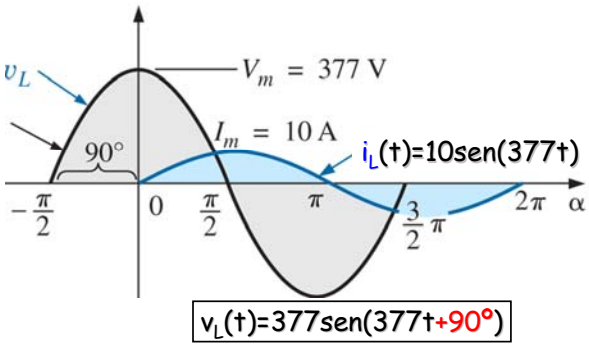
$i_L(t)=10\text{sen}(377t)$ $\quad V_m=I_mX_L$ $\quad X_L=\omega L$

$I_m=10A$; $\omega=377\text{rad/s}$

$X_L=(377\text{rad/s})(0.1H)=37.7\ \Omega$

$V_m=(10A)(37.7\Omega)=377V$

$v_L(t)=377\text{sen}(377t+90°)$



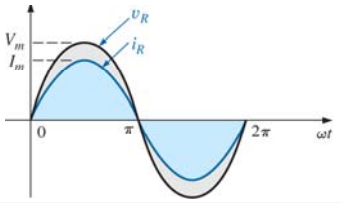
$v_L(t)=377\text{sen}(377t+90°)$

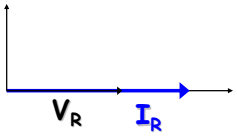
Notación Fasorial:

$y(t)=Y_m\text{sen}(\omega t+\varphi)$ $\quad \Rightarrow \boldsymbol{Y} = Y_{Eficaz}\angle\varphi°$

$v_R(t)=V_m\text{sen}(\omega t)$ $\quad \Rightarrow \boldsymbol{V_R} = V_R\angle 0°$

$i_R(t)=I_m\text{sen}(\omega t)$ $\quad \Rightarrow \boldsymbol{I_R} = I_R\angle 0°$





$$\boldsymbol{V_R} = V_R\angle 0°$$

Ley de Ohm: $\boldsymbol{I} = \dfrac{V\angle 0°}{R\angle\varphi_R} =$

φ_R:0→ $\boldsymbol{I} = \dfrac{V\angle 0°}{R\angle 0°} = \dfrac{V}{R}\angle(0°-0°) = \dfrac{V}{R}\angle 0°$

$$\boldsymbol{Z_R} = R\angle 0°$$

Impedancia Resistiva

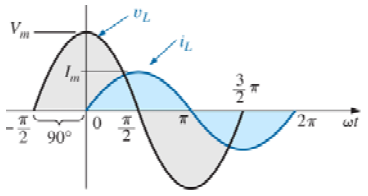
Circuito Inductivo:

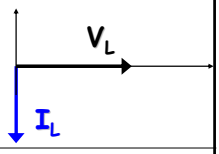
$v_L(t)=V_m\text{sen}(\omega t)$ → $\boldsymbol{V_L} = V\angle 0°$

Ley de Ohm: $\boldsymbol{I_L} = \dfrac{V_L\angle 0°}{X_L\angle\varphi_L} = \dfrac{V_L}{X_L}\angle(0°-\varphi_L)$ $\quad \varphi_L=90°$

$\boldsymbol{I_L} = \dfrac{V_L}{X_L}\angle(0°-90°) = \dfrac{V_L}{X_L}\angle -90°$ $\quad \boldsymbol{I_L} = I\angle -90°$

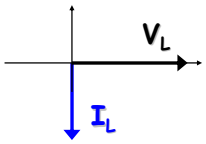
$\boldsymbol{Z_L} = X_L\angle 90°$



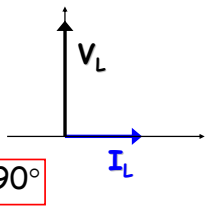


Circuito Inductivo:

$\boldsymbol{V_L} = V\angle 0°$ $\quad \boldsymbol{I_L} = I\angle -90°$



$\boldsymbol{V_L} = V\angle 90°$ $\quad \boldsymbol{I_L} = I\angle 0°$



$\boldsymbol{Z_L} = X_L\angle 90°$

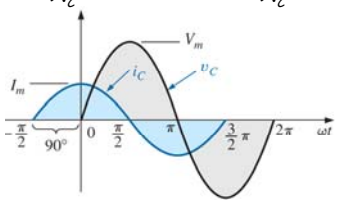
Circuito Capacitivo:

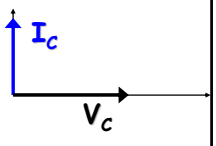
$v_C(t)=V_C\text{sen}(\omega t) \rightarrow \quad V_c = V\angle 0°$

Ley de Ohm: $I_c = \dfrac{V_C\angle 0°}{X_C\angle \varphi_C} = \dfrac{V_C}{X_C}\angle(0° - \varphi_C)$ $\quad \varphi_C$=-90°

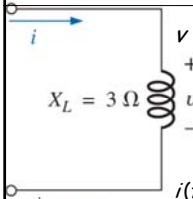
$I_c = \dfrac{V_C}{X_C}\angle(0° - (-90°)) = \dfrac{V_C}{X_C}\angle 90°$ $\quad I_c = I\angle 90°$

$Z_C = X_C\angle -90°$





Ejemplo: Utilizando álgebra compleja determine la corriente del circuito. Grafique.

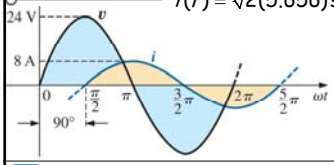


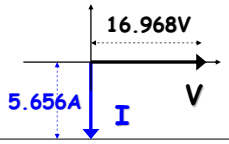
$v = 24\text{sin}(\omega t) \Rightarrow V = \dfrac{24}{\sqrt{2}}\angle 0° = 16.968\angle 0°$

$I = \dfrac{V}{Z_L} = \dfrac{V\angle 0°}{X_L\angle 90°} = \dfrac{16.968\angle 0°}{3\Omega\angle 90°} =$

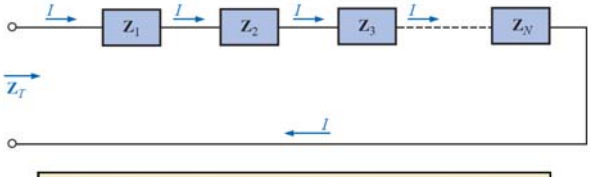
$I = 5.656A\angle -90°$

$i(t) = \sqrt{2}(5.656)\text{sin}(\omega t - 90°) = 8\text{sin}(\omega t - 90°)$



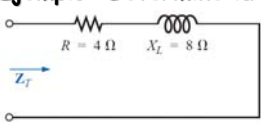


Impedancias en Serie:



$$Z_T = Z_1 + Z_2 + Z_3 + \cdots + Z_N$$

Ejemplo: Determine la Z_T:



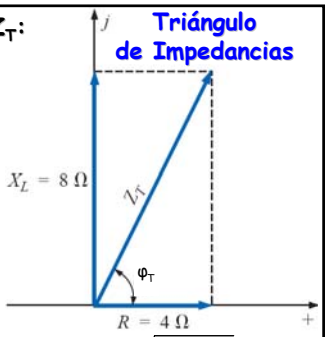
$Z_T = Z_1 + Z_2$

$= R\angle 0° + X_L\angle 90°$

$= R + jX_L = 4\ \Omega + j8\ \Omega$

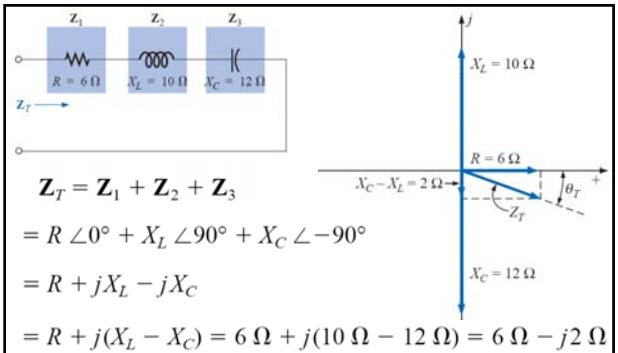
$Z_T = 8.944\ \Omega\angle 63.43°$

Triángulo de Impedancias



$|Z_T| = \sqrt{R^2 + X_L^2}$

$\varphi_T = Arctg\dfrac{X_L}{R}$



$Z_T = Z_1 + Z_2 + Z_3$

$= R\angle 0° + X_L\angle 90° + X_C\angle -90°$

$= R + jX_L - jX_C$

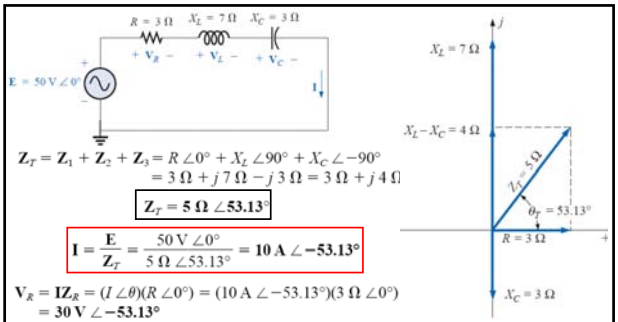
$= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega$

$Z_T = 6.325\ \Omega\angle -18.43°$

Ley de Ohm en AC:

$$I = \frac{V}{Z}$$

OJO!!! V e I son FASORES!!!



$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R\angle 0° + X_L\angle 90° + X_C\angle -90°$$
$$= 3\ \Omega + j7\ \Omega - j3\ \Omega = 3\ \Omega + j4\ \Omega$$

$$\mathbf{Z}_T = 5\ \Omega\angle 53.13°$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V}\angle 0°}{5\ \Omega\angle 53.13°} = 10\text{ A}\angle -53.13°$$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle 0°) = (10\text{ A}\angle -53.13°)(3\ \Omega\angle 0°)$$
$$= 30\text{ V}\angle -53.13°$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\angle\theta)(X_L\angle 90°) = (10\text{ A}\angle -53.13°)(7\ \Omega\angle 90°)$$
$$= 70\text{ V}\angle 36.87°$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\angle\theta)(X_C\angle -90°) = (10\text{ A}\angle -53.13°)(3\ \Omega\angle -90°)$$
$$= 30\text{ V}\angle -143.13°$$

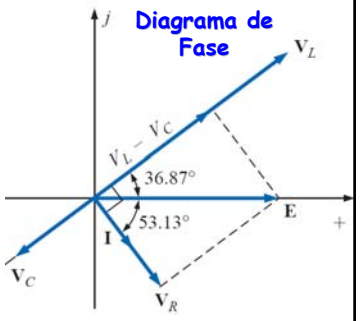
$\boldsymbol{V} = 50\angle 0°$

$\boldsymbol{I} = 10\angle -53.13°$

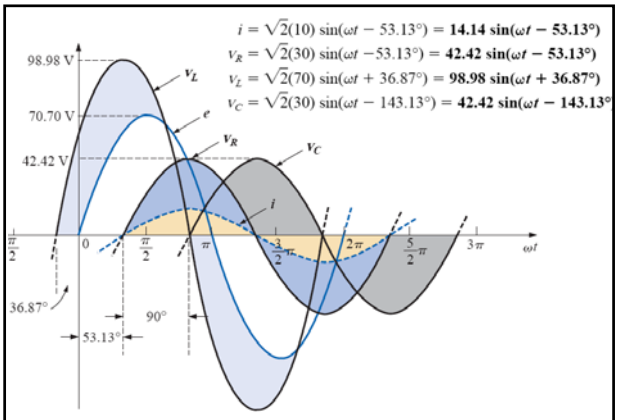
$\boldsymbol{V_R} = 30\angle -53.13°$

$\boldsymbol{V_L} = 70\angle 36.87°$

$\boldsymbol{V_C} = 30\angle -143.13°$

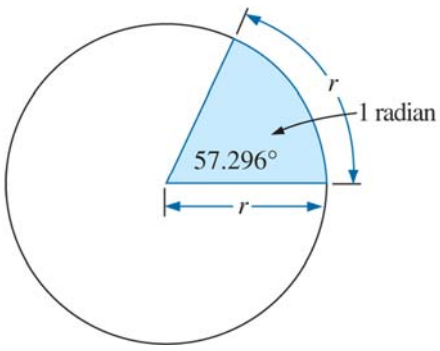


$$\Sigma_{\circlearrowleft}\,\mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L - \mathbf{V}_C = 0$$

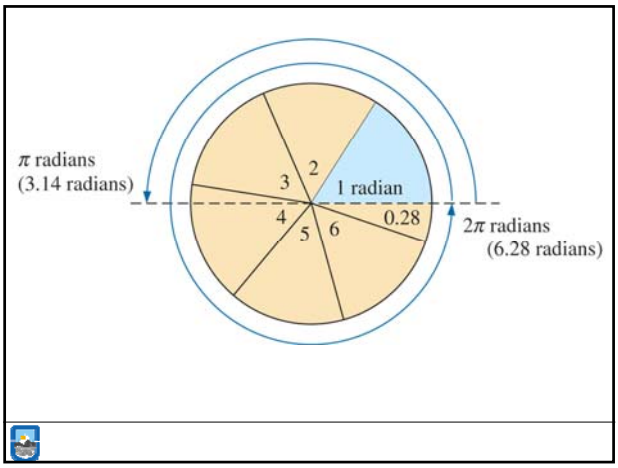


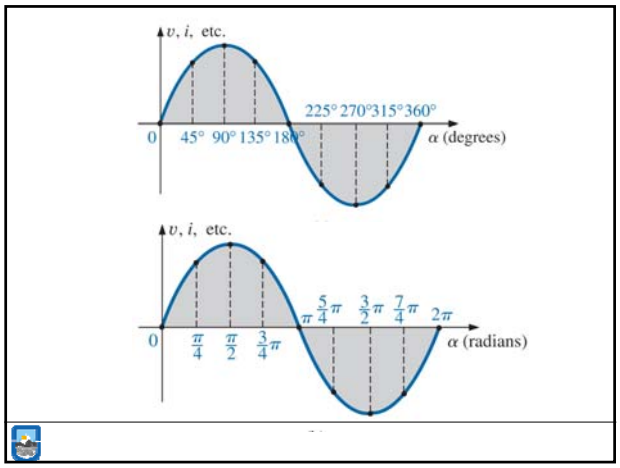
Miércoles 23: Laboratorio 3
Diferencia de Fase Circuito Serie RC

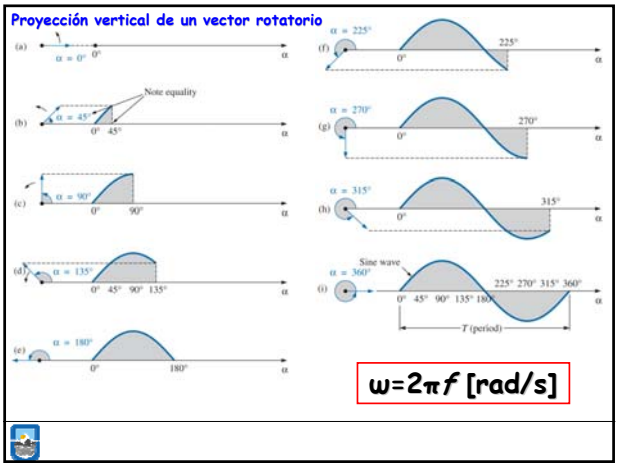
Repaso

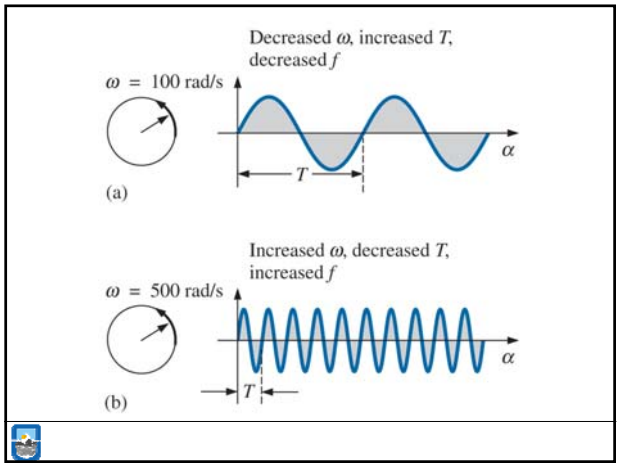


2πrad=360°


π radians
(3.14 radians)
1 radian
0.28
2π radians
(6.28 radians)
1
2
3
4
5
6


v, i, etc.
0
45° 90° 135° 180°
225° 270° 315° 360°
α (degrees)
v, i, etc.
0
π/4 π/2 3/4 π
π 5/4 π 3/2 π 7/4 π 2π
α (radians)


Proyección vertical de un vector rotatorio
Note equality
Sine wave
T (period)
ω=2πf [rad/s]


Decreased ω, increased T, decreased f
ω = 100 rad/s
T
α
(a)
Increased ω, decreased T, increased f
ω = 500 rad/s
T
α
(b)